

Encyclopedia of World Biography on Leonard Euler

The Swiss mathematician Leonhard Euler (1707-1783) made important original contributions to every branch of mathematics studied in his day.

The son of a clergyman, Leonhard Euler, was born in Basel on April 15, 1707. He graduated from the University of Basel in 1724. In 1727 Catherine I invited him to join the Academy of Sciences in St. Petersburg, Russia; he became professor of physics in 1730 and professor of mathematics in 1733. In 1741 Frederick the Great called him to Berlin. Euler was director of mathematics at the Academy of Sciences there until 1766, when he returned to St. Petersburg, as director of the academy. Soon after his return to St. Petersburg, Euler became blind but continued to dictate books and papers. In 1776, having lost his first wife, he married his sister-in-law. Euler died in St. Petersburg on Sept. 7, 1783.

Analysis and the Calculus

Euler's textbooks presented all that was known of mathematics in a clear and orderly manner, setting fashions in notation and method which have been influential to the present day. At various times he used the notations $f(x)$, e , i , though he was not in every case the first to do so. Angles of a triangle he represented by A , B , C and the corresponding sides by a , b , c , thus simplifying trigonometric formulas. Moreover, he defined the trigonometric values as ratios and introduced the modern notation.

Euler's first great textbook was *Introductio in analysin infinitorum* (1748). The first volume is devoted to the theory of functions, and in particular the exponential, logarithmic, and trigonometric functions. These functions are developed as infinite series.

At this time no clear notion of convergence existed; it is not surprising, therefore, that although Euler warned against the use of divergent series, he himself did not always succeed in avoiding such series. He also resolved the subtle problem of the logarithms of negative and imaginary numbers, and he proved that e is irrational.

The second volume of the *Introductio* contains an analytical study of curves and surfaces. First Euler considered the general equation of the second degree in two dimensions, showing that it represents the various conic sections; the discussion included a treatment of asymptotes, centers of curvature, and curves of higher degree. Turning to the case of three dimensions, Euler gave the first complete classification of surfaces

represented by the general equation of the second degree. This part of the *Introductio* really constituted the first treatise on analytical geometry.

Euler wrote two great textbooks on the calculus: *Institutiones calculi differentialis* (1755) and *Institutiones calculi integralis* (3 vols., 1768-1770). His outstanding achievement in this field was the invention of the calculus of variations, described in *The Art of Finding Curves Which Possess Some Property of Maximum or Minimum* (in Latin; 1744). This subject grew out of the isoperimetric problems which had created great interest among the mathematicians of the time. Such problems, involving the determination of the form of a curve having a certain maximum or minimum property, were quite different from the ordinary maximum and minimum problems of the differential calculus. Although particular problems had been solved by others, it was Euler who developed a general method. His method was essentially geometrical, and this made the solution of the simpler problems very clear.

Theory of Numbers

Another of Euler's outstanding textbooks was *Vollständige Anleitung zur Algebra* (1770). The first volume takes algebra up to cubic and biquadratic equations, while the second is devoted to the theory of numbers.

Euler proved many of the results that had been stated by Pierre Fermat. Fermat's most famous proposition, the general proof of which has defeated the efforts of the ablest mathematicians to the present day, states that the equation $x^n + y^n = z^n$ has no solution in integers for n greater than 2. Euler made the first attack on the problem, demonstrating the theorem for $n = 3$ and $n = 4$.

Fermat also stated that the Diophantine equation $x^2 - ay^2 = 1$ always has an infinity of solutions. Although Euler failed to prove this assertion, he used successive solutions of the equation to compute approximations to a and, reversing the procedure, found solutions of the equation by developing a as a continued fraction.

Mechanics and Hydrodynamics

Euler's *Mechanica* (1736) was the first textbook in which Newtonian particle dynamics was developed using analytical methods. Another of Euler's works, *Theoria motus corpus solidorum seu rigidorum* (1765), treated the mechanics of solid bodies in the same way; by resolving the motion of a solid body into a motion of the center of mass and a rotation

about this point, Euler arrived at the general equations of motion. The term "moment of inertia" was here introduced for the first time. A memoir presented by Euler to the Paris Academy of Sciences in 1755 contained an even greater achievement, namely, the general equations of hydrodynamics, consisting of the equation of continuity, expressing the constancy of mass of a fluid element and the effects of pressure on it as it moves along, and the equations of motion, relating the forces to the acceleration of the fluid element.

Euler also devoted much attention to the problems of astronomy. He studied the attraction of ellipsoids and recognized that the tides are effectively generated by the horizontal components of the disturbing forces. Euler also contributed to the three-body problem, which was important in the theory of the moon's motion.

Euler's general knowledge was extensive. Like his contemporary Jean d'Alembert, he wrote a book on the theory of music. His interests also extended to mathematical puzzles, optics, the theory of heat, and acoustics. Finally, Euler attempted a popularization of philosophy and natural science in *Lettres à une princesse d'Allemagne*, a work in three volumes which comprises his full range of scientific endeavors.

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