Encyclopedia of World Biography on Euclid

The Greek mathematician Euclid (active 300 BC) wrote the *Elements,* a collection of geometrical theorems. The oldest extant major mathematical work in the Western world, it set a standard for logical exposition for over 2,000 years.

Virtually nothing is known of Euclid personally. It is not even known for certain whether he was a creative mathematician himself or was simply good at compiling the work of others. Most of the information about Euclid comes from Proclus, a 5th-century-AD. Greek scholar. Because Archimedes refers to Euclid and Archimedes lived immediately after the time of Ptolemy I, King of Egypt (ca. 306-283 BC), Proclus concludes they were contemporaries. Euclid's mathematical education may well have been obtained from Plato's pupils in Athens, since it was there that most of the earlier mathematicians upon whose work the *Elements* is based had studied and taught.

No earlier writings comparable to the *Elements* of Euclid have survived. One reason is that Euclid's *Elements* superseded all previous writings of this type, making it unnecessary to preserve them. This makes it difficult for the historian to investigate those earlier mathematicians whose works were probably more important in the development of Greek mathematics than Euclid's. About 600 B.C. the Greek mathematician Thales is said to have discovered a number of theorems that appear in the *Elements*. It might be noted too that Eudoxus is also given credit for the discovery of the method of exhaustion, whereby the area of a circle and volume of a sphere and other figures can be calculated. Book XII of the *Elements* makes use of this method. Although mathematics may have been initiated by concrete problems, such as determining areas and volumes, by the time of Euclid mathematics had developed into an abstract construction, an intellectual occupation for philosophers rather than scientists.

The *Elements*

The *Elements* consists of 13 books. Within each book is a sequence of propositions or theorems, varying from about 10 to 100, preceded by definitions. In Book I, 23 definitions are followed by five postulates. After the postulates, five common notions or axioms are listed. The first is, "Things which are equal to the same thing are also equal to each other." Next are 48 propositions which relate some of the objects that were defined and which lead up to Pythagoras's theorem: in right-angled triangles the square on the side subtending the right angle is equal to the sum of the squares on the sides containing the right angle. The usual elementary course in Euclidean geometry is based on Book I.

The remaining books, although not so well known, are more advanced mathematically. Book II is a continuation of Book I, proving geometrically what today would be called algebraic identities, such as $(a + b)^2 = a^2 + b^2 + 2ab$, and generalizing some propositions of Book I. Book III is on circles, intersections of circles, and properties of tangents to circles. Book IV continues with circles, emphasizing inscribed and circumscribed rectilinear figures.

Book V of the *Elements* is one of the finest works in Greek mathematics. The theory of proportions discovered by Eudoxus is here expounded masterfully by Euclid.

The theory of proportions is concerned with the ratios of magnitudes (rational or irrational numbers) and their integral multiples. Book VI applies the propositions of Book V to the figures of plane geometry. A basic proposition in this book is that a line parallel to one side of a triangle will divide the other two sides in the same ratio.

As in Book V, Books VII, VIII, and IX are concerned with properties of (positive integral) numbers. In Book VII a prime number is defined as that which is measured by a unit alone (a prime number is divisible only by itself and 1). In Book IX proposition 20 asserts that there are infinitely many prime numbers, and Euclid's proof is essentially the one usually given in modern algebra textbooks. Book X is an impressively well-finished treatment of irrational numbers or, more precisely, straight lines whose lengths cannot be measured exactly by a given line assumed as rational.

Books XI-XIII are principally concerned with three-dimensional figures. In Book XII the method of exhaustion is used extensively. The final book shows how to construct and circumscribe by a sphere the five Platonic, or regular, solids: the regular pyramid or tetrahedron, octahedron, cube, icosahedron, and dodecahedron.

Manuscript translations of the *Elements* were made in Latin and Arabic, but it was not until the first printed edition, published in Venice in 1482, that geometry, which meant in effect the *Elements,* became important in European education. The first complete English translation was printed in 1570. It was during the most active mathematical period in England, about 1700, that Greek mathematics was studied most intensively. Euclid was admired, mastered, and utilized by all major mathematicians, including Isaac Newton.

The growing predominance of the sciences and mathematics in the 18th and 19th centuries helped to keep Euclid in a prominent place in the curriculum of schools and universities throughout the Western world. But also the *Elements* was considered educational as a primer in logic.

Euclid's Other Works

Some of Euclid's other works are known only through references by other writers. The *Data* is on plane geometry. The word "data" means "things given." The treatise contains 94 propositions concerned with the kind of problem where certain data are given about a figure and from which other data can be deduced, for example: if a triangle has one angle given, the rectangle contained by the sides including the angle has to the area of the triangle a given ratio.

On Division (of figures), also on plane geometry, is known only in the Arabic, from which English translations were made. Proclus refers to it when speaking of dividing a figure into other figures different in kind, for example, dividing a triangle into a triangle and a quadrilateral. *On Division* is concerned with more general problems of division. As an example, one problem is to draw in a given circle two parallel chords cutting off between them a given fraction of the area of the circle.

The *Conics* appears to have been lost by the time of the Greek astronomer Pappus (late 3d century A.D.). It is frequently referred to by Archimedes. As the name suggests, it dealt with the conic sections: the ellipse, parabola, and hyperbola, to use the names given them later by Apollonius of Perga.

A work which has survived is *Phaenomena*. This is what today would be called applied mathematics; it is about the geometry of spheres applicable to astronomy. Another applied work which has survived is the *Optics*. It was maintained by some that the sun and other heavenly bodies are actually the size they appear to be to the eye. This work refuted such a view by analyzing the relationship between what the eye sees of an object and what the object actually is. For example, the eye always sees less than half of a sphere, and as the observer moves closer to the sphere the part of it seen is decreased although it appears larger.

Another lost work is the *Porisms,* known only through Pappus. A porism is intermediate between a theorem and a problem; that is, rather than something to be proved or something to be constructed, a porism is concerned with bringing out another aspect of something that is already there. To find the center of a circle or to find the greatest common divisor of two numbers are examples of porisms. This work appears to have been more advanced than the *Elements* and perhaps if known would give Euclid a higher place in the history of mathematics.